

# Causal Inference

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## Reference on History of Causal Inference

Holland, Paul W. 1986. "Statistics and Causal Inference." *Journal of the American Statistical Association* 81(396): 945-960.

# What is a Causal Effect?

$$Y_i(1) - Y_i(0)$$

where  $Y_i(1) = Y_i(T_i = 1)$  for some treatment variable  $T$ .

$Y_i(1)$  and  $Y_i(0)$  are *potential outcomes* in that they represent the outcomes for individual  $i$  had they received the treatment or control respectively.

The **fundamental problem of causal inference** is that only one of  $Y_i(1)$  and  $Y_i(0)$  is observed, so we can never find the true causal effect.

The approach we will discuss is known as the *Rubin Causal Model*.

Instead of the individual treatment effect, we might be interested in the **average treatment effect (ATE)**:

$$E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)]$$

We cannot find the ATE because of the unobserved potential outcomes.

We might also be interested in the **average treatment effect on the treated (ATT)**:

$$E[Y(1|T = 1) - Y(0|T = 1)] = E[Y_t(1) - Y_t(0)]$$

We cannot find the ATT because of **unobserved** potential outcomes.

## Causal Inference as Missing Data Problem

$i$	$T_i$	$Y_i(0)$	$Y_i(1)$	$Y_i(1) - Y_i(0)$
1	0	3	5	2
2	1	2	5	3
3	1	5	4	-1
4	0	2	7	5
5	1	1	2	1

Some potential outcomes are **unobserved**, as are the ATE and ATT.

$$\text{ATE} = 2$$

$$\text{ATT} = 1$$

## Estimating ATE

We can estimate the ATE in the following way:

$$\begin{aligned}\hat{\text{ATE}} &= E[Y_t(1) - Y_c(0)] \\ &= E[Y_t(1)] - E[Y_c(0)]\end{aligned}$$

Both quantities are observed.

We basically find the average  $Y$  for observations that received treatment and average  $Y$  for observations that received control.

What assumptions do we need for this estimate to be unbiased?

- ▶ SUTVA
- ▶ unconfoundedness/ignorability

# Stable Unit Treatment Value Assumption

The **stable unit treatment value assumption (SUTVA)** assumes that

- ▶ the treatment status of any unit does not affect the potential outcomes of the other units (non-interference)
- ▶ the treatments for all units are comparable (no variation in treatment)

Violations:

- ▶ Job training for too many people may flood the market with qualified job applicants (interference)
- ▶ Some patients get extra-strength aspirin (variation in treatment)

# Ignorability/Unconfoundedness

Unconfoundedness (strong ignorability):

$$(Y(1), Y(0)) \perp T$$

Treatment assignment is independent of the outcomes ( $Y$ ).

Ignorability and Unconfoundedness are often used interchangeably. Technically, unconfoundedness is a stronger assumption. Most people just say ignorability.

Violations:

- ▶ Omitted Variable Bias



# Classical Randomized Experiment

The gold standard of scientific research.

1. Randomly sample units from population.
2. Randomly assign treatment and control to the units.
3. Estimate ATE.

SUTVA: can theoretically control for treatment variation and non-interference

Ignorability: controlled for by random treatment assignment

## Possible problems:

- ▶ Compliance to treatment assignment:
  - ▶ Never-taker: Unit never takes treatment
  - ▶ Always-taker: Unit always takes treatment
  - ▶ Complier: Unit takes treatment when assigned and control when not assigned
  - ▶ Defier: Unit takes treatment when not assigned and control when assigned
  
- ▶ Unlucky random treatment assignment violates ignorability
  - ▶ Problem goes away as sample size gets large.

## Observational Data

We have a dataset where we only observe after the experiment occurred and we have no control over treatment assignment.

This is the case with most of the sciences.

1. Gather dataset.
2. Estimate ATE or ATT with a model.

SUTVA: assumed (a problematic assumption most of the time)

Ignorability: include covariates to get conditional ignorability

$$(Y(1), Y(0)) \perp T | X$$

Treatment assignment is independent of the outcomes ( $Y$ ) given covariates  $X$ .

## Problems:

- ▶ SUTVA assumption
- ▶ Omitted variable bias
  - ▶ Don't include all the variables that makes treatment assignment independent of  $Y$ .
- ▶ Model Dependence
  - ▶ We try to alleviate the curse of dimensionality and problem of continuous covariates by specifying a model.
  - ▶ Estimates of ATE or ATT may differ depending on the model you specify.

## Matching to Ameliorate Model Dependence

If we had pairs of observations that had the exact same covariate values (perfect **balance**) and differed only on treatment assignment, then we would have perfect conditional ignorability (assuming no omitted variable bias).

Then we will get the same results regardless of the model.

Matching is a method of trying to achieve better balance on covariates and reduce model dependence.

Goal: **BALANCE** on covariates

## Matching on Propensity Scores

Suppose each observation has some true probability of receiving the treatment.

- ▶ A doctor examines a patient and has a probability of giving the patient a drug, depending on the patient's age, health, etc.

The probability of receiving the treatment is the **propensity score**.

We don't know the true propensity score but we can estimate it for each observation with a regression of  $T$  on  $X$  (assuming we have the right set of  $X$  that went into the decision for assigning treatment).

Then we match an observation that received treatment with an observation with a similar propensity score that received control.

Since we don't have the true propensity scores, we need to check for balance on our covariates at the end.

## Other Matching Algorithms

Matching on propensity scores is only one way of matching.

Other ways include:

- ▶ Matching on Mahalanobis distances
- ▶ Genetic Matching
- ▶ Coarsened Exact Matching (CEM)

# Instrumental Variables

Goal: Estimate Causal Effects

Problem in Observational Data: Non-ignorability of treatment assignment (and SUTVA)

Solution so far: Include covariates and match

Another solution: **Instrumental Variables**

The idea: Find an instrument  $Z$  that is randomly assigned (or assignment is ignorable) and that affects  $Y$  only through  $T$ .

Example:  $Y$  = post-Vietnam War civilian mortality;  $T$  = serving in the military during Vietnam War;  $Z$  = draft lottery

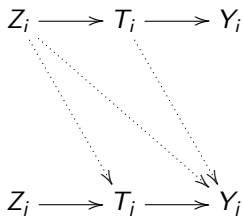


# The Potential Outcomes Approach

Assumptions:

1. SUTVA:  $Z_i$  does not affect  $T_j$  and  $Y_j$  and  $T_i$  does not affect  $Y_j$  for all  $i \neq j$  (non-interference) and there is no variation in the treatment or the instrument.

Figure: SUTVA Assumption implies absence of dotted arrows.



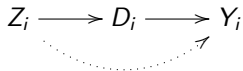
*Example: The veteran status of any man at risk of being drafted in the lottery was not affected by the draft status of others at risk of being drafted, and, similarly, that the civilian mortality of any such man was not affected by the draft status of others.*

## 2. Random (Ignorable) Assignment of the Instrument $Z$

*Example: Assignment of draft status was random.*

## 3. Exclusion Restriction: Any effect of $Z$ on $Y$ must be via an effect of $Z$ on $T$ .

**Figure:** Exclusion assumption implies absence of dotted arrow.



*Example: Civilian mortality risk was not affected by draft status once veteran status is taken into account.*

## 4. Nonzero Average Causal Effect of $Z$ on $T$ .

*Example: Having a low lottery number increases the average probability of service.*

## 5. Monotonicity: No Defiers

*Example: There is no one who would have served if given a high lottery number, but not if given a low lottery number.*

If all the assumptions hold, then the **Local Average Treatment Effect (LATE)** of  $T$  on  $Y$  is

$$\text{LATE} = \frac{\text{Effect of Z on Y}}{\text{Effect of Z on T}}$$

It is only a local average treatment effect because it's the effect of  $T$  on  $Y$  for the subpopulation of compliers, and not the whole population.

Angrist, Joshua D., Guido W. Imbens and Donald B. Rubin. 1996.  
"Identification of Causal Effects Using Instrumental Variables."  
*Journal of the American Statistical Association* 91(434):444-455.

Describes instrumental variables in more detail and compares it to the econometric treatment.