Discrete Distributions

- Poisson Distribution

$$\begin{aligned} \theta &\sim \text{Poisson}(\lambda);\ \lambda > 0 \\ p(\theta) &= \frac{\exp(-\lambda)\lambda^\theta}{\theta!} \text{ for } \theta = 0, 1, 2, \ldots \\ E(\theta) &= \lambda \\ \text{Var}(\theta) &= \lambda \end{aligned}$$

- Binomial Distribution

$$\begin{aligned} \theta &\sim \text{Binomial}(n, p);\ p \in [0, 1] \\ p(\theta) &= \binom{n}{\theta} p^\theta (1-p)^{n-\theta} \text{ for } \theta = 0, 1, 2, \ldots, n \\ E(\theta) &= np \\ \text{Var}(\theta) &= np(1-p) \end{aligned}$$

- Multinomial Distribution

$$\begin{aligned} \boldsymbol{\theta} &\sim \text{Multinomial}(n; p_1, \cdots, p_k);\ p_j \in [0, 1],\ \sum_{j=1}^{k} p_j = 1 \\ p(\boldsymbol{\theta}) &= \left(\frac{n!}{\theta_1! \cdots \theta_k!}\right) p_1^{\theta_1} \ldots p_k^{\theta_k} \text{ for } \theta_j = 0, 1, 2, \ldots, n;\ \sum_{j=1}^{k} \theta_j = n \\ E(\theta_j) &= np_j \\ \text{Var}(\theta_j) &= np_j(1-p_j) \end{aligned}$$

Continuous Distributions

- Uniform Distribution

$$\begin{aligned} \theta &\sim \text{Uniform}(\alpha, \beta);\ \beta > \alpha \\ p(\theta) &= \frac{1}{\beta - \alpha} \text{ for } \theta \in [\alpha, \beta] \\ E(\theta) &= \frac{\alpha + \beta}{2} \\ \text{Var}(\theta) &= \frac{(\beta - \alpha)^2}{12} \end{aligned}$$

- Normal Distribution

$$\begin{aligned}\theta &\sim \mathrm{N}(\mu, \sigma^2);\ \sigma^2 > 0 \\ p(\theta) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\theta-\mu)^2}{2\sigma^2}\right) \\ E(\theta) &= \mu \\ \mathrm{Var}(\theta) &= \sigma^2\end{aligned}$$

- Multivariate Normal Distribution

$$\begin{aligned}\boldsymbol{\theta} &\sim \mathrm{N}_d(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \\ p(\boldsymbol{\theta}) &= (2\pi)^{-d/2}|\boldsymbol{\Sigma}|^{-1/2} \exp\left(-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu})\right) \\ E(\boldsymbol{\theta}) &= \boldsymbol{\mu} \\ \mathrm{Var}(\boldsymbol{\theta}) &= \boldsymbol{\Sigma}\end{aligned}$$

- Gamma Distribution

$$\begin{aligned}\theta &\sim \mathrm{Gamma}(\alpha, \beta);\ \alpha, \beta > 0 \\ p(\theta) &= \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta};\ \theta > 0 \\ E(\theta) &= \frac{\alpha}{\beta} \\ \mathrm{Var}(\theta) &= \frac{\alpha}{\beta^2}\end{aligned}$$

- Inverse-gamma Distribution

$$\begin{aligned}\theta &\sim \text{Inv-Gamma}(\alpha, \beta);\ \alpha, \beta > 0 \\ p(\theta) &= \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{-(\alpha+1)}e^{-\beta/\theta};\ \theta > 0 \\ E(\theta) &= \frac{\beta}{\alpha-1} \text{ for } \alpha > 1 \\ \mathrm{Var}(\theta) &= \frac{\beta^2}{(\alpha-1)^2(\alpha-2)},\ \alpha > 2\end{aligned}$$

- Exponential Distribution

$$\begin{aligned}\theta &\sim \mathrm{Exponential}(\lambda);\ \lambda > 0 \\ p(\theta) &= \lambda e^{-\lambda\theta};\ \theta > 0 \\ E(\theta) &= \frac{1}{\lambda} \\ \mathrm{Var}(\theta) &= \frac{1}{\lambda^2}\end{aligned}$$

- Beta Distribution

$$\begin{aligned} \theta &\sim \text{Beta}(\alpha, \beta);\ \alpha, \beta > 0 \\ p(\theta) &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1};\ \theta \in [0,1] \\ E(\theta) &= \frac{\alpha}{\alpha+\beta} \\ \text{Var}(\theta) &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \end{aligned}$$

- Dirichlet Distribution

$$\begin{aligned} \boldsymbol{\theta} &\sim \text{Dirichlet}(\alpha_1, \dots, \alpha_k);\ \alpha_j > 0 \\ p(\boldsymbol{\theta}) &= \frac{\Gamma(\alpha_1 + \cdots + \alpha_k)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)}\theta_1^{\alpha_1-1}\cdots\theta_k^{\alpha_k-1};\ \theta_1, \dots, \theta_k \geq 0,\ \sum_{j=1}^{k}\theta_j = 1 \\ E(\theta_j) &= \frac{\alpha_j}{\alpha_0} \text{ where } \alpha_0 \equiv \sum_{j=1}^{k}\alpha_j \\ \text{Var}_\text{j}(\theta) &= \frac{\alpha_j(\alpha_0 - \alpha_j)}{\alpha_0^2(\alpha_0+1)} \\ \text{Cov}(\theta_i, \theta_j) &= -\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)} \end{aligned}$$

Formulas

- (Univariate) Change of Variables formula

  Let $Y = g(X)$ and $X = g^{-1}(Y)$. Then

$$p_Y(y) = p_X(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right|$$

Conjugacy

Likelihood	Prior	Posterior
$Y_i \sim \text{Bernoulli}(\pi)$	$\pi \sim \text{Beta}(\alpha, \beta)$	$\text{Beta}(\alpha + \sum_{i=1}^n y_i, \beta + n - \sum_{i=1}^n y_i)$
$Y_i \sim \text{Binomial}(N, \pi)$	$\pi \sim \text{Beta}(\alpha, \beta)$	$\text{Beta}(\alpha + \sum_{i=1}^n y_i, \beta + nN - \sum_{i=1}^n y_i)$
$Y_i \sim \text{Poisson}(\lambda)$	$\lambda \sim \text{Gamma}(\alpha, \beta)$	$\text{Gamma}(\alpha + \sum_{i=1}^n y_i, \beta + n)$
$Y_i \sim \text{Multinomial}(N, \boldsymbol{\pi})$	$\boldsymbol{\pi} \sim \text{Dirichlet}(\alpha_1, \dots, \alpha_k)$	$\text{Dirichlet}(\alpha_1 + \sum_{i=1}^n y_{i1}, \dots, \alpha_k + \sum_{i=1}^n y_{ik})$